Total No. of Printed Pages—8

1 TDC MTH COM (S)

2009

MATHEMATICS (Speciality)

Full Marks : 70
Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. *(a)* *(i)* If $pqr = 1$, show that

$$\frac{1}{1+p+q^{-1}} + \frac{1}{1+q+r^{-1}} + \frac{1}{1+r+p^{-1}} = 1$$ 5

(ii) If α and β are the roots of the equation $x(x-3) = 4$, then find out the value of $\alpha^2 + \beta^2$. 5

(iii) The length of a line segment whose end points are $A(2, -3)$ and $B(10, y)$ is 10 cm. Find the value of y. 4

Or

(b) *(i)* Solve : 2

$$2^{x+3} + 2^{x+1} = 320$$

(ii) In how many ways a team of 11 players can be selected out of 15, if (1) one particular player must be there in each team and (2) one particular player is excluded from each team? 5

(iii) Show that the lines $2x + 3y + 7 = 0$ and $3x - 2y + 5 = 0$ are perpendicular to each other. 2

(iv) Find the intercepts that the line $5x - 3y - 7 = 0$ makes on the axes. What is the slope of this line? 4+1

2. (a) (i) Define odd and even function with examples. 1½×2=3

(ii) If $f(x) = 2x^3 - 3x + 1$, then find $f(-3)$. 2

(iii) Evaluate : 5

$$\lim_{x \to \infty} (\sqrt{x^2 + 5x + 4} - \sqrt{x^2 - 3x + 4})$$

(iv) Find the value of

$$\int \frac{x^2\, dx}{1 + 2x^3}$$ 2

(v) Find the value of

$$\int_1^2 (x+1)^2 \, dx$$ 2

Or

(b) (i) A function $f(x)$ is defined as

$$f(x) = -x \quad , \text{ when } x \leq 0$$
$$= x \quad , \text{ when } 0 < x < 1$$
$$= 2 - x \, , \text{ when } x \geq 1$$

show that it is continuous at $x = 0$. 3

(ii) If $y = (x+2)(x+1)^2$, find the value of $\frac{dy}{dx}$. 3

(iii) The total cost function for producing x units of a commodity is $TC = 60 - 12x + 2x^2$. Find the level of output at which TC is minimum. 5

(iv) If $u = \frac{y}{z} + \frac{z}{x} + \frac{x}{y}$, prove that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$$ 3

3. (a) (i) Distinguish between determinants and matrices. 2

(ii) Prove by using properties

$$\begin{vmatrix} 1 & 1^2 & 2^2 \\ 2 & 2^2 & 4^2 \\ 3 & 3^2 & 6^2 \end{vmatrix} = 0$$

2

(iii) Construct 2×3 matrix whose elements a_{ij} are given by

$$a_{ij} = \frac{(i+j)^2}{2}$$

3

(iv) Find x and y, if

$$\begin{bmatrix} 1 & x+y \\ x-y & 0 \end{bmatrix} = \begin{bmatrix} 1 & 7 \\ 1 & 0 \end{bmatrix}$$

2

(v) Solve by matrix method, the equations

$$x + 2y = 1$$
$$3x + y = 4$$

5

Or

(b) (i) Define minors and cofactors of the elements of a determinant. 2

(ii) Prove that

$$\begin{vmatrix} 1+a & b & c \\ a & 1+b & c \\ a & b & 1+c \end{vmatrix} = 1 + a + b + c$$

5

(iii) Find A and B, if

$$A + B = \begin{bmatrix} 4 & 1 & -1 \\ 9 & 2 & 7 \\ 3 & -1 & 4 \end{bmatrix} \text{ and}$$

$$A - B = \begin{bmatrix} 2 & -3 & 5 \\ -1 & 2 & 3 \\ 1 & 1 & 2 \end{bmatrix}$$ 5

(iv) If

$$A = \begin{bmatrix} 2 & 3 \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 3 & 0 \\ -2 & 0 \end{bmatrix}$$

find AB. 2

4. *(a)* *(i)* What do you understand by Linear Programming Problem? Give a general mathematical formulation for the linear programming problem. 2+3

(ii) Discuss briefly the application of linear programming in business and commerce. 4

(iii) Solve the following LPP using graphical method : 5

Maximize $Z = 3x_1 + 4x_2$

subject to

$$x_1 + x_2 \leq 450$$
$$2x_1 + x_2 \leq 600$$
$$x_1, x_2 \geq 0$$

Or

(b) *(i)* Write short note on limitation of LPP. 4

(ii) What are slack and surplus variables? 2

(iii) Use simplex method to

Maximize $Z = 6x_1 + 8x_2$

subject to

$$5x_1 + 10x_2 \leq 60$$
$$4x_1 + 4x_2 \leq 40$$
$$x_1, x_2 \geq 0$$

8

5. *(a)* *(i)* Fill up the gap : 1

Characteristic of log 0·0032 is ——.

(ii) Evaluate : 2

$$\log_2 [\log_{\sqrt{2}} (\log_3 81)]$$

(iii) If $\frac{\log x}{y-z} = \frac{\log y}{z-x} = \frac{\log z}{x-y}$, then prove that

$$x^x y^y z^z = 1$$

3

(iv) The population of a country increase every year by 2·4% of the population at the beginning of that year. In what time will the population double itself? 4

(v) A man deposits Rs 1,200 at a bank at the end of each year at 5% compound interest. What amount will be deposited to his account at the end of 15 years? 4

Or

(b) (i) Find the compound interest on Rs 16,000 @ 5% p.a. for $2\frac{1}{2}$ years, if the interest is calculated half-yearly. 4

(ii) A sinking fund is created for the replacement of a machine after 25 years; whose present cost is Rs 80,000. How much money should be provided out of profits, each year, for the fund, if the investment can earn interest at 10% p.a. and the cost of the machine at the time of replacement is 20% more than the present cost? 5

(iii) Find the value of x, if

$$x = \log_{0.5} 0.25$$ 1

(iv) Simplify : 2

$$\log_2 \sqrt{6} + \log_2 \sqrt{\frac{2}{3}}$$

(v) If $2\log a + 3\log b - 2 = 0$, then prove that $a^2 b^3 = 100$. 2

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